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Participants giving a talk

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Stability of Switched Systems for Random Switching Functions: A Survey

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By a switched system, we mean a family of continuous–time dynamical systems and a rule that determines at each time which dynamical system is responsible of the time evolution. More precisely, let $\{f_u : u \in U\}$ (where U is a subset of R^m, $m \in N$) be a finite or infinite set of sufficiently regular vector fields on a manifold M, and consider the family of dynamical systems: $\dot{x} = f_u(x), \quad x \in M$. The rule is given by assigning the so-called switching function, i.e. a measurable function $u(.) : [0, \infty[\to U \subset \mathbb{R}^m$. Here, we consider the situation in which the switching function is not known a priori and represents some phenomenon (e.g. a disturbance) that is not possible to control. A typical problem for switched systems goes as follows. Assume that, for every $u \in U$, the dynamical system $\dot{x} = f_u(x)$ satisfies a given property (P). Then one can investigate conditions under which property (P) still holds for $\dot{x} = f_{u(t)}(x)$, where $u(.)$ is an arbitrary switching function. In this talk I will consider the problem of the asymptotic stability and I will make a survey of some recent results.

Stability of elastic systems

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Stability and stabilization by introducing fast oscillations into a system are widely studied in literature. Basic example is stabilization of equilibrium of inverted pendulum by means of fast harmonic oscillation of its suspension point, [1]. Methods of high-order averaging developed in [3] allow to study stability and asymptotic stability of a wider class of time-variant systems. As an illustration a stabilization condition for pendulum under arbitrary (fast) oscillation of its suspension point has been established. An extension of this work onto the case of double inverted pendulum was done in [2]. Now we propose to deal with a wider class of systems: the class of elastic systems described by means of a partial differential equations.

References

[1] V.I. Arnold.Mathematical Methods of Classical Mechanics. Graduate Texts in Mathe- matics. Springer-Verlag, 2 edition, 1989.

[2] M.I. Caiado and A.V. Sarychev. Remarks on stability of inverted pendula (unpublished). 2003.

[3] A.V. Sarychev. Stability criteria for time-periodic systems via high-order averaging tech- niques.Nonlinear Control in year 2000, 2: 365-377, 2001.

Normality in Necessary Conditions for Dynamic Optimization Problems with Inequality Constraints

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There has been a longstanding interest in deriving conditions under which dynamic optimization problems are normal, that is, the necessary conditions of optimality can be written with the multiplier associated with the objective function positive. The study of these normality conditions is relevant either for problems in which the terminal state is constrained or in which the trajectory is constrained. Most results in the literature address the former case while the trajectory constrained case has been subject of less attention. Here, we address the latter situation. Our work builds upon previous results on nondegenerate conditions for trajectory constrained problems to provide, in some situations, even further information to select minimizers. We start by discussing a result on normality for optimal control problems and then explore its application in the context of Calculus of Variations. A normal form of the necessary conditions for problems of the calculus of variations with inequality constraints is established. The special structure the Calculus of Variations problems permits the derivation of Constraint Qualifications that are much easier to verify than in the optimal control case.

Review of generalized differentials

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In control theory one very often has to deal with nonsmooth functions and with set-valued maps. In both cases classical derivatives do not exist. Generalized differentials, like Clarke generalized Jacobians, Warga derivate containers or Sussmann generalized differential quotients, make it possible to differentiate such maps in order, for example, to use them in maximum principle. But it is well known that there are many theories of generalized differentials and almost every such theory leads to different results. Our goal is to review some of them, to compare those that are comparable and to show relations among them. We will focus only on these theories that have applications in control theory.

Adaptive algorithms for Wiener nonlinear systems and their application to neuromuscular blockade control

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Wiener models consisting of a linear dynamic element followed in series by a static nonlinear element are considered to be ideal for representing a wide range of nonlinear processes behaviour. Several authors have established utility of this model structure in representing anaesthesia patient response to drug administrations. Since patients differ in metabolism, pre-existing medical conditions and surgical procedures, individualized models must be established for each patient with limited information and data measurements. This paper develops a real-time identification method that incorporates expert information and real time data to improve control system accuracy. The model parameters are first classified from knowledge available in literature and also from clinical data. This initial model is then improved using real-time data. An model reference adaptive control scheme for Wiener type nonlinear systems is derived. Global stability of the proposed scheme is established when the nonlinearity can be represented by an multisegment piecewise-linear function. The control of neuromuscular blockade is used as a case study. This work was supported by the Ministério da Educação/Fundo Social Europeu, under the program PRODEP III.

Generalized synthesis for singular nonlinear control systems

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Generalized (discontinuous) Hamiltonian flows can be used to obtain a geometrical description of the generalized extremal synthesis for certain types of control systems wich are affine with respect to control. We discuss some pheneomena that arise in these synthesis and discuss some ways to extend the idea of generalized Hamiltonian flows to some types of fully nonlinear systems and/or affine systems with high order of singularity.

Fitting Geodesics on Data in $SO(n)$

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The main objective is to find geodesics on the Lie group of special orthogonal matrices $SO(n)$ fitting a given data set of points. We start with an optimization problem in the Euclidean space $\mathbb{R}^n$ whose solution is a straight line fitting a given data set of points. Afterwards we present the natural generalization of this problem to the Lie group $SO(n)$ and derive the corresponding Euler-Lagrange equations. An interesting thing that occurs is that when the given data set of points belongs to a connected and compact abelian subgroup of $SO(n)$ the geodesic that best fits the given points passes through their Riemannian center of mass.

Nonstandard proofs of some Critical Point Theorems

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We will present nonstandard proofs of some theorems in Critical Point Theory, namely, we will prove the Mountain Pass Theorem for coercive functionals defined if finite dimensional real Banach spaces, using arguments of discrete type.

On the Existence of a Common Polynomial Lyapunov Function for Linear Switched Systems

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We consider linear switched systems $\dot{x}(t) = A_{u(t)}x(t)$, $x \in \mathbb{R}^n$, $u \in U$, and the problem of global exponential stability for arbitrary switching functions, uniform with respect to switching (**GUES** for short). We first prove that, given a **GUES** system, it is always possible to build a common polynomial Lyapunov function. Then our main result is that the degree of that polynomial Lyapunov function cannot be bounded over all the **GUES** systems. This result answers a question raised by Dayawansa and Martin.

On observability conditions of row–finite countable systems of linear and non-linear differential equations

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We study here linear systems which are naturally described by infinite matrices. We restrict our studies to systems that are described by row-finite matrices whose rows contain only finitely many elements different from zero. We admit matrices with rows and columns indexed by arbitrary countable sets without any specified order. Simple examples show that such systems may have infinitely many smooth solutions corresponding to the same initial condition. On the other hand no analytic solution may exist. We choose then formal solutions defined by formal power series. The main problem studied here is observability. Thus we extend the system of differential equations adding the output equations. Observability may be defined in the standard way. We show that a necessary and sufficient condition for injectivity is existence of a row-finite left inverse of the Kalman matrix. This is equivalent to the fact that any state variable can be expressed as a linear combination of *finitely* many outputs and their derivatives. *Local* observability of *nonlinear* infinite systems of differential equations with output is studied. The essential part of those investigations was producing a good definition of local observability. Such system is locally observable at a state x^0 if locally around x^0 for each state variable x_i we can deduce that $x_i = x_i^0$ from *finitely* many equations of the form $\varphi(x) = \varphi(x^0)$, where φ is a function from the observation algebra of the system. If the system is linear than local and global observability coincide.

Control of vibrations in some hybrid systems of controlled machine units

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Hybrid systems of controlled machine units are widespread in heavy, extractive and manufacturing industry and vibrations in such systems always have negative influence on systems functioning. Therefore the problem is to find possibilities of vibrations control. The mathematical model is a non-classical boundary value problem in PDE where the second boundary condition is a non-linear one. The complexity of the boundary value problem results in impossibility to find its exact solution. Therefore both numerical and asymptotical methods are used, which allow determining frequencies of possible vibrations. The asymptotical method allows also determining amplitudes of vibrations. The conditions at which vibrations of different character take place in the considered hybrid systems are defined. Influence of different parameters of controlled machine units on vibrations frequencies and amplitudes is analyzed. For the purpose of undesirable vibrations control it is proposed to use different feedbacks in the considered hybrid systems.

Newton's problem of the body of minimal resistance

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A body moves with constant velocity through a rarefied medium of immovable particles. It is required to determine the shape of body minimizing resistance of the medium to the motion of body. This problem was solved by Newton in the class of convex axisymmetric bodies of fixed length and width. In the last decade the interest to this problem and its generalizations revived; the problem was solved for various classes of bodies, and new unexpected results were obtained. These results will be discussed in the talk:

- solutions in classes of non-convex and/or non-symmetric bodies;
- estimation of resistance for bodies performing both translational and rotational motion;
- motion in a medium of positive temperature.

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Towards sufficient conditions for the local optimality of a bang-singular Pontryagin extremal

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We consider an optimal control problem with a Mayer cost $c_0(\xi(0)) + c_f(\xi(T_f)) \to \min$. The control system is single-input control-affine: $\dot{\xi}(t) = f_0(\xi(t)) + uf_1(\xi(t))$ and the control u takes values in $[-1,1]$. The state space and the end-point constraints are smooth manifolds, f_0 and f_1 are smooth vector fields. We assume $\widehat{u}$ is a bang-singular control i.e. there exists $T_s \in (0, T_f)$ such that $\widehat{u}(t) \equiv 1 \quad t \in [0, T_s]\,, \quad \widehat{u}(t) \in (-1,1) \quad t \in (T_s, T_f]\,.$ Let $\widehat{\xi}$ be a trajectory associated to $\widehat{u}$ and satisfying the end-point constraints. We assume $(\widehat{\xi}, \widehat{u})$ is a Pontryagin extremal of the problem. Let $\widehat{\lambda}(\cdot)$ be the covector associated to $(\widehat{\xi}, \widehat{u})$ in Pontryagin maximum principle. Since $\widehat{u}$ is bang-singular we have $\langle \widehat{\lambda}(t), f_1(\widehat{\xi}(t)) \rangle > 0 \quad t \in (0, T_s) \qquad \langle \widehat{\lambda}(t), f_1(\widehat{\xi}(t)) \rangle = 0 \quad t \in (T_s, T_f)\,.$

We are interested in finding first order and second order sufficient conditions for $(\widehat{\xi}, \widehat{u})$ to be a state-local minimizer of the problem. Our hypotheses will concern the positivity of an extended second variation on the singular arc and the Hamiltonians associated to some iterated Lie brackets of the vector fields f_0, f_1, in particular we assume the strengthened generalized Legendre condition holds. A first step in studying this problem is to study the optimality of a bang arc (i.e. $\widehat{u} \equiv 1$ on $[0, T_f]$) such that $\langle \widehat{\lambda}(T_f), f_1(\widehat{\xi}(T_f)) \rangle = 0$.

Eigenvectors and eigenvalues of graphs with regularity constraints

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Graphs with (k,τ)-regular sets are examples of graphs with regularity constraints. A (k,τ)-regular set of a graph G is a subset of vertices $S \subseteq V(G)$ inducing a k-regular subgraph and such that each vertex not in S has τ neighbors in S. The (k,τ)-regular sets can model several problems. For example, if a simple graph (i.e, a graph with no loops nor parallel edges) has a Hamiltonian cycle then its line graph has a $(2,4)$-regular set or, equivalently, if a simple graph is Hamiltonian then the line graph of its subdivision (that is, the graph obtained after the insertion of a vertex in each edge) has a $(2,2)$-regular set. Another example comes from the relations between the existence of certain matchings in a simple graph and the existence of (k,τ)-regular sets in its line graph. Namely, a simple graph G has a perfect matching if and only if its line graph has a $(0,2)$-regular set, also G has a perfect induced matching (which is an induced matching covering all the edges) if and only if its line graph $L(G)$ has a $(0,1)$-regular set. The spectral properties of the adjacency and Laplacian matrices of these graphs can help to recognize these sets. Furthermore, basic properties of regular graphs with (k,τ)-regular sets are presented. Namely, we conclude that if the 4^{th} Moore graph exists and has a maximum independent set with 400 vertices then it has no (k,τ)-regular sets with $k-\tau = 7$. Additionally, if G is a primitive strongly regular graph with parameters $(n,p;a,c)$ and adjacency matrix A_G, and $\lambda \in \sigma(A_G) \setminus \{k-\tau, p\}$, then $S \subset V(G)$ is (k,τ)-regular if and only if $\forall v \in Ker(A_G - \lambda I)$, $\sum_{j \in S} v_j = 0$. Furthermore, if S is (k,τ)-regular then $k = \frac{c\lambda(|S|-1)+pc}{\lambda(p-\lambda)} - \frac{p}{\lambda}$ and $\tau = \frac{c}{p-\lambda}|S|$.

Iterative predictive control for batch processes

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The aim of this talk is to present an optimal predictive controller for batch processes. Predictive control makes use of an objective function $J = \sum_{i=1}^{p} \hat{e}\,(t+i|t) + \sum_{i=0}^{m-1} u\,(t+i)$ that is optimized in order to obtain the control law. Using past batches information (ie. trajectories followed by the system in the last runs) improves controller performance. A linear time varying model, which is different for each batch, can be a good approximation of the nonlinear system. Therefore the controller is applied to nonlinear batch plants. It also considers the possibility of adding constraint to the control problem.

Formal Control Theory and Applications

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We will present some formal tools and non-associative algebras that play an important role in the combinatoric representation of flow maps. Coordinate maps and formal flows will be then applied to Lyapunov stability and integrability of sub-Riemannian geodesics.

On the generation of spline curves using geometric algorithms

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A new geometric algorithm to generate C^k-smooth spline curves, which interpolate a given set of data (points and velocities) on a complete Riemannian manifold, is discussed. This new algorithm is compared with other classical interpolating schemes such as the classical De Casteljau procedure. The new approach is first presented in the Euclidean case in order to observe its main properties and motivate its generalization.

An Example of the Lavrentiev Phenomenon in 2D

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This work provides an example of regular problem of the Calculus of Variations in $\mathbb{R}^2$, which exhibits the $W^{1,\infty} - W^{1,p}$ Lavrentiev gap.

Starting with the example of Maniá we extend it to two dimensions. Nontrivial questions arise in what regards Lebesgue sets of functions from $W^{1,\infty}$ and of projections of these sets. Accomplishing this study we arrive to a class of functionals whose integrands are convex and coercive with respect to gradient and which exhibit the Lavrentiev phenomenon.

On the Classical Newton's Problem of Minimal Resistance

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In the celebrated *Principia Mathematica*, published in 1686, Isaac Newton propose what is now called *the Newton's Problem of Minimal Resistance*. The problem consists of determining the shape of a revolution body which offers minimum resistance when it is moving, with constant velocity and along its symmetry axis, in a resistant medium. Newton has considered the situation: (i) of a moving body on "a rare medium" – the particles of the medium are all equal, are equally spaced, and do not interact each other; (ii) when the particles of the medium are immovable; (iii) when the collisions with the body are perfectly elastic. Newton has not only proposed the problem, but also indicate in his book what the correct solution to his problem is. He has not explained, however, how such solution can be obtained. In this talk we shall present the classical (old) mathematical formulation of Newton's problem, and show that such mathematical problem fails to satisfy all the classical necessary optimality conditions of the calculus of variations. We conclude that the old mathematical formulation of the problem has no solution, and that such formulation is not well posed. We then proceed to the correct formulation of Newton's problem of minimal resistance (an optimal control problem), showing how one can solve it with the help of the Pontryagin Maximum Principle of Optimal Control Theory.

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[2]This work is part of the author's MSc project, which is being developed at the University of Aveiro under the scientific supervision of Delfim F. M. Torres.

Pricing Digital Options with Transaction Costs and an Interval Market Model

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We investigate a two player zero-sum differential game $\min_\xi \max_\tau J(x,\tau,\xi)$ in a $3D$ plus time space arising in mathematical finance [1]. The dynamics $\dot{x} = f(x,\tau,\xi)$ are given by $x = (u,v,w)$,

$$\left\{\begin{array}{lllll} \dot{u} &=& \tau u & & \text{u : underlying stock price} \\ \dot{v} &=& \tau v + \xi & \text{or} \quad v(t_k^+) = v(t_k^-) + \xi_k & \text{v : value of the investement in underlying stock} \\ \dot{w} &=& \tau v - C(\xi) & \text{or} \quad w(t_k^+) = w(t_k^-) - C(\xi_k) & \text{w : worth of the hedging portfolio} \end{array}\right.$$

and the criteria by

$$J = \left[N(u(T)) - \int_0^T (\tau(t)v(t) - C(\xi(t)))dt + \sum_k C(\xi_k) \right].$$

The transaction costs $C(\xi(t))$ are chosen proportional to the amount of the transaction ξ and thus characterize by a positive piecewise affine function : $C(\xi(t)) = \left\{ \begin{array}{l} C^-\xi(t) \text{ if } \xi(t) \leq 0 \text{ with } C^- < 0 \\ C^+\xi(t) \text{ if } \xi(t) \geq 0 \text{ with } C^+ < 0 \end{array}\right.$. For digital options, the terminal cost function $N(u(T))$ is supposed to be discontinuous while it was convex in the previous works [1],[2]. The control $t \mapsto \xi(t)$, which represents the trading strategy, may contain impulses; and the other control, the rate of change of the stock price $t \mapsto \tau(t)$, is supposed to be bounded ($\tau(t) \in [\tau^-, \tau^+]$ with $\tau^- < 0$ and $\tau^+ > 0$). The objective is to determine the value function $W(t,u,v)$ which is a viscosity solution [3] of the Isaacs equation [1]

$$0 = \min \left\{ \frac{\partial W}{\partial t} + \max_{\tau \in [\tau^-,\tau^+]} \tau [\frac{\partial W}{\partial u} u + (\frac{\partial W}{\partial v} - 1)v] \; , \frac{\partial W}{\partial v} + C^+ \; , -(\frac{\partial W}{\partial v} + C^-) \right\} \; . \qquad (1)$$

We investigate two ways to solve this problem : a geometric analysis of the trajectories with tools of semipermeability where the impulses are trajectories orthogonal to the (t,u) plane; and an analytic way where we exhibit a function via a representation theorem, and show that it is a discontinuous (the uniqueness of the solution is not proved yet) viscosity solution of (1).

References

[1] P. Bernhard, N. El Farouq and S. Thiery. *An impulse differential game arising in finance with interesting singularities.* 10th ISDG International Symposium on Dynamic Games and Applications. S^T Petersburg July 8-11, 2002.

[2] P. Bernhard. *An explicit viscosity solution of an Isaacs quasi variationnal inequality arising in option pricing.* 4th ISDG Workshop. Goslar, Germany, May 19-21, 2003.

[3] G. Barles *Solutions de viscosité des équations de Hamilton-Jacobi.* Mathématiques et Applications, 17. Springer-Verlag, Paris, 1994.

Lipschitzian Regularity of the Minimizing Trajectories in the Calculus of Variations and Optimal Control: a Survey

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Under standard hypotheses of the Tonelli existence theory in the Calculus of Variations, the existence of minimizers is guaranteed in the class of absolutely continuous functions possibly with unbounded derivative. As it is known, in such cases the optimality conditions – like the Euler-Lagrange equation – may fail. Therefore it is important to try to obtain Lipschitzian regularity conditions under which the minimizers are Lipschitzian. Main part of the results obtained (starting with those of Leonida Tonelli) refer to the Basic Problem of the Calculus of Variations. Less is known for the problems of the calculus of variations with high-order derivatives and for the problems of optimal control. In this talk I will give a survey on Lipschitzian Regularity conditions for the Minimizing Trajectories in the Calculus of Variations and Optimal Control. Some open problems needing further study will be presented.

References

[1] Delfim F. M. Torres, Lipschitzian Regularity of the Minimizing Trajectories for Nonlinear Optimal Control Problems, *Mathematics of Control, Signals, and Systems*, Vol. 16, 2003, pp. 158–174.

[2] Delfim F. M. Torres, Carathéodory-Equivalence, Noether Theorems, and Tonelli Full-Regularity in the Calculus of Variations and Optimal Control, *Journal of Mathematical Sciences*, Vol. 120, No. 1, 2004, pp. 1032–1050.

[3] Delfim F. M. Torres, The Role of Symmetry in the Regularity Properties of Optimal Controls, *Proceedings of Institute of Mathematics of National Academy of Sciences of Ukraine*, Vol. 50, Part 3, pp. 1488–1495, 2004.

Numerical methods in optimal control

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There exist two main types of numerical methods in optimal control: direct methods, which reduce the problem to a constrained nonlinear optimization problem (or nonlinear programming problem); and indirect methods, based on the Maximum Principle, which reduce the problem to a shooting problem. We will review these numerical methods, discuss their efficiency and make some comparisons, and give some examples and applications.

Observers for locally observable systems

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An observer is the system whose task is state estimation on the basis of the known inputs and outputs of the original system. Assuming local observability we allow for the existence of indistinguishable states in the state space so the reconstruction of the unknown state is impossible but we can reconstruct the whole class of states that are indistinguishable. Such observers, that produce an estimation of the indistinguishable class of unknown state, are called multiobservers. Thus a multiobserver is a system, whose input is the output of the original system and whose output is a multivalued map with values in $\mathbb{R}^n$. A construction of multiobservers, conditions of their existance and their properties are given.

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